1-1

Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

$t\ (0, \infty)$

$V\ (4, \infty)$

b. Find the initial velocity of the object.

$V(0) = 0 - 0 + 4 = 4$

c. Find the acceleration function $a(t)$ of the object at any time t.

$a(t) = \frac{\partial}{\partial t} V(t)$

$a(t) = 24t - \frac{1}{2}(t^{-1/2})$

d. Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.

$V(t) = \frac{\partial}{\partial t} S(t)$

$S(t) = \frac{12t^3}{3} - \frac{2}{3}t^{(3/2)} + 4t + 1$

e. Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.

minimum (4)

maximum $(16 - \sqrt{1})$

f. Find the average velocity of the object on the interval $[0, 1]$.

$\int_0^1 V(t) = \frac{12t^3}{3} - \frac{2}{3}t^{(3/2)} + 4t + 1$

$\frac{12}{3} - \frac{2}{3} 1^{(3/2)} + 4 + 1 \quad - \quad \frac{0}{3} - 0 + 0 + 1$

$4 - \frac{2}{3} + 5$

$9 - \frac{2}{3} \quad - \quad 1 \quad = \frac{22}{3}$

1-2

Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

limitation on variables t & V

greater than 0

4

b. Find the initial velocity of the object.

$v(0) = 12(0)^2 - \sqrt{(0)} + 4 = 4$

c. Find the acceleration function $a(t)$ of the object at any time t.

$a(t) = 24t - \frac{1}{2}t^{-\frac{1}{2}}$

d. Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.

$s(t) = \int v(t)\,dt$

e. Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.

$V(0) = 12(0^2) - \sqrt{0} + 4 = 4$

$v(1) = 12(1^2) - \sqrt{1} + 4 = 15$

$24(0) - \frac{1}{2}(0^{-\frac{1}{2}}) = 0$

$24(1) - \frac{1}{2}(1^{-\frac{1}{2}}) = 23.5$

f. Find the average velocity of the object on the interval $[0, 1]$.

$\frac{s(1) - s(0)}{1 - 0}$

1-3

Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

starts at 0, doesn't end

b. Find the initial velocity of the object.

$v(0) = 4$

c. Find the acceleration function $a(t)$ of the object at any time t.

$$\frac{d}{dt} v(t) = \frac{d}{dt}\, 12t^2 - \sqrt{t} + 4$$
$$= 24t - 1/2\, t^{-1/2}$$

d. Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.

$$s(t) = \int 12t^2 - \underset{t^{1/2}}{\sqrt{t}} + 4 \, dt = 12\frac{t^3}{3} - \frac{t^{3/2}}{3/2} + 4t + 1$$
$$= 4t^3 - \frac{2t^{3/2}}{3} + 4t + 1$$

e. Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.

min. 4
max 15

f. Find the average velocity of the object on the interval $[0, 1]$.

$$\frac{15-4}{1} = 11$$

1-4

Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

$V \geq 4$

$t \geq 0$

b. Find the initial velocity of the object.

$V(0) = 12(0)^2 - \sqrt{0} + 4 = 4$

c. Find the acceleration function $a(t)$ of the object at any time t.

$a(t) = 24t - \frac{1}{2}t^{-\frac{1}{2}}$

d. Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.

$S(t) = \int 12t^2 - \sqrt{t} + 4\,dt = 4\frac{t^3}{3} - \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + 4t$

$S(t) = 4\frac{t^3}{3} - \frac{2}{3} \cdot t^{\frac{3}{2}} + 4t$

e. Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.

$V(0) = 12(0)^2 - \sqrt{0} + 4 = 4$ Max $= 15$

$V(1) = 12(1)^2 - \sqrt{1} + 4 = 15$ Min $= 4$

f. Find the average velocity of the object on the interval $[0, 1]$.

$S(0) = 4\frac{(0)^3}{3} - \frac{2}{3} \cdot (0)^{\frac{3}{2}} + 4(0) = 0$

$S(1) = 4\frac{(1)^3}{3} - \frac{2}{3} \cdot (1)^{\frac{3}{2}} + 4(1) = \frac{14}{3}$

$\frac{\frac{14}{3} - 0}{1 - 0} = \frac{14}{3}$

1-5

Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

$t \geq 0$

$v > 0$

b. Find the initial velocity of the object.

$v(0) = 4$

c. Find the acceleration function $a(t)$ of the object at any time t.

$a(t) = 24t - t^{\frac{1}{2}}$

d. Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.

$s(t) = \int 12t^2 - \sqrt{t} + 4\,dt = \frac{12t^3}{3} - \frac{t}{2} + 4t$

$s(0) = \frac{12(0)^3}{3} - 0 + 4(0)$

$= 0$

e. Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.

f. Find the average velocity of the object on the interval $[0, 1]$.

$\left(\frac{12(1)^3}{3} - \frac{1}{2} + 4\right) - \left(\frac{12(0)^3}{3} - \frac{0}{2}\right) = \frac{15}{2}$

$s(t) = \int 12t^2 - \sqrt{t} + 4\,dt = 4t^3 - \frac{t^{3/2}}{3/2} + 4t + 1$

1-6

Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

$t = 0$

$V \leq 15$ ft/s

b. Find the initial velocity of the object.

Initial velocity is 4 ft/sec

$V(0) = 12(0)^2 - \sqrt{0} + 4 = 4$

c. Find the acceleration function $a(t)$ of the object at any time t.

$d/dt = 24t - \frac{1}{2}(t)^{-1/2}$

$a(t) = 24t - \frac{1}{\sqrt{t}}$

d. Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.

$s(t) = \int 12t^2 - \sqrt{t} + 4\,dt = \boxed{4t^3 - \frac{t^{3/2}}{3/2} + 4t + 1}$

e. Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.

$v(0) = 12(0)^2 - \sqrt{0} + 4 = 4$

$V(1) = 12(1)^2 - \sqrt{1} + 4 = 15$

min velocity of 4 ft/sec

max velocity of 15 ft/sec

f. Find the average velocity of the object on the interval $[0, 1]$.

$\frac{s(1) - s(0)}{1 - 0}$

$(4(1)^3 - 1^{3/2} \cdot 2/3 + 4(1) + 1) - (4(0)^3 - 0^{3/2} \cdot 2/3 + 4(0) +$

$4 - 2/3 + 4 + 1 - 1$

$4 - 2/3 + 4 + 1 - 1$

Average velocity $= 22/3$ ft/s

$12t^2 - \sqrt{t} + 4 = 0 - 4$

$12t^2 - \sqrt{t} = -4$

Q 8. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$, [redacted] of an object in motion. Show all work clearly.

$t^{1/2 - 1}$

a. State the limitations that are placed on variables t and V in context to the problem. (2 points)

$t \neq$ negative ; $\sqrt{-t} = \text{DNE}$

b. Find the initial velocity of the object. (2 points)

$24t - \frac{1}{\sqrt{t}}$

c. Find the acceleration function a(t) of the object at any time t. (2 points)

$24t - \frac{1}{\sqrt{t}} = 0$

$t = 0.1202$

d. Find the position function s(t) of the object at any time t, where $s(0) = 1$. (2 points)

$24t - \frac{1}{\sqrt{t}} = 1$

$\sqrt{24t} - 1 = 1 + 1$

$\sqrt{24t}^2 = 2^2$

$24t = 4$

$t = 4$

e. Find the maximum and the minimum velocity of the object on the interval $[0, 1]$. (2 points)

$V(0) = 4$ $(0, 4)$

$V(1) = 15$ $(1, 15)$

f. Find the average velocity of the object on the interval $[0, 1]$. (2 points)

$= 7.33$

2-2

Q 8. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$, [redacted] of an object in motion. Show all work clearly.

$12t^2 - t^{1/2} + 4$

a. State the limitations that are placed on variables t and V in context to the problem. (2 points)

t should be bigger than 0

b. Find the initial velocity of the object. (2 points)

$24t - \frac{1}{2}t^{-1/2} = 23.5\ m/s = V_i$

used 1 in substitution of t

c. Find the acceleration function a(t) of the object at any time t. (2 points)

normal acceleration is $9.8\ m/s^2$

d. Find the position function s(t) of the object at any time t, where $s(0) = 1$. (2 points)

$x = x_i + at^2$

$= 0m + 9.8(1)^2 = 9.8m$

e. Find the maximum and the minimum velocity of the object on the interval $[0, 1]$. (2 points)

max velocity: $23.5\ m/s$

min: DNE or $4\ m/s$ taken into account of 4 in the function?

f. Find the average velocity of the object on the interval $[0, 1]$. (2 points)

$v - v_i = \frac{x - x_i}{2} t = 28.4$

$v - (23.5) = 4.9$

$+ 23.5$

2-3

Q 8. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$, [redacted] of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem. (2 points)

t = time

V = velocity

b. Find the initial velocity of the object. (2 points)

$V(t) = 12t^2 - t^{1/2} + 4 = 0$

$12t^2 - t^{1/2} = -4$

c. Find the acceleration function a(t) of the object at any time t. (2 points)

$V'(t) = 24t - \frac{1}{2t^{1/2}} = a(t)$

d. Find the position function s(t) of the object at any time t, where $s(0) = 1$. (2 points)

$S(t) = 24t - \frac{1}{2t^{1/2}} = 1$

$S(0) = 24(0) - \frac{1}{2(0)^{1/2}} = 0$

e. Find the maximum and the minimum velocity of the object on the interval $[0, 1]$. (2 points)

$12(1)^2 - \sqrt{1} + 4 = 15$ max

$12(0)^2 - \sqrt{0} + 4 = 4$ min

f. Find the average velocity of the object on the interval $[0, 1]$. (2 points)

max 15
min 4 average : 9.5

2-4

Q 8. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$, ~~where $0 \le t$~~ of an object in motion. Show all work clearly.

$= 12t^2 - t^{1/2} + 4$

a. State the limitations that are placed on variables t and V in context to the problem. (2 points)

b. Find the initial velocity of the object. (2 points)

$12t^2 - t^{1/2} + 4 = 0$ $\qquad t^2 - t^{1/2} = \frac{1}{3}$ $\qquad t = 0{,}577$

$\frac{12t^2}{n} - t^{1/2} = \frac{4}{n}$ $\qquad t^4 = \frac{1}{9}$

c. Find the acceleration function a(t) of the object at any time t. (2 points)

d. Find the position function s(t) of the object at any time t, where $s(0) = 1$. (2 points)

$12(0)^2 - 0^{1/2} + 4 = 4$

e. Find the maximum and the minimum velocity of the object on the interval $[0, 1]$. (2 points)

f. Find the average velocity of the object on the interval $[0, 1]$. (2 points)

3-1

(12 points) 10. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion.

a) State the limitations that are placed on variables t and V in context to the problem.

t has to be positive

b) Find the initial velocity of the object.

$V(0) = 12(0)^2 - \sqrt{0} + 4$ initial velocity $= 0$

c) Find the acceleration function a(t) of the object at any time t.

$a(t) = \dfrac{12t^2 - \sqrt{t} + 4}{t}$

d) Find the position function s(t) of the object at any time t, where s(0) =1.

$s(t) = 24t - \frac{1}{2}t^{-1/2} + 1$

e) Find the maximum and the minimum velocity of the object on the interval [0, 1].

$V(0) = 0$, minimum

$V(1) = 15$, maximum

f) Find the average velocity of the object on the interval [0, 1].

$\dfrac{0 + 15}{2} = \dfrac{15}{2}$

(10 points) 11. a) Sketch the regions bounded by the x-axis and the graph of $y = x^2 - 2x - 3$ on the interval from x=0 to x=5.

b) Label the intercepts and the point(s) of intersection.

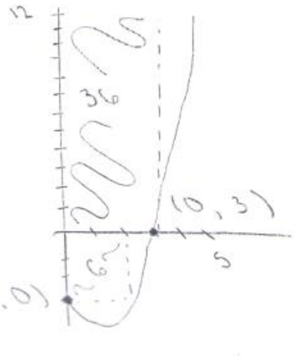
c) Then find the total area of the bounded regions.

$0 = (x-3)(x+1)$

$x = 3 \qquad x = -1$

$\int_0^5 (x^2 - 2x - 3)\,dx$

$\frac{x^3}{3} - x^2 - 3x \Big|_0^5$



$\frac{5^3}{3} - 5^2 - 3(5) - 0$

$= \frac{125}{3} - 25 - 15$

$= \frac{125}{3} - 40$

$= \frac{5}{3} + 36 + 6 = \frac{131}{3}$

3-2

(12 points) 10. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion.

a) State the limitations that are placed on variables t and V in context to the problem.

t cannot be negatie as time will alway move forward

V is always in respect to t

$t^{\frac{1}{2}}$

$\frac{1}{2} t^{\frac{1}{2}}$

b) Find the initial velocity of the object.

$V(0) = 12(0)^2 - \sqrt{0} + 4 \quad V(0) = 0 - 0 + 4 \quad V(0) = 4$

c) Find the acceleration function a(t) of the object at any time t.

$a(t) = V'(t)$

$a(t) = 24t - \frac{1}{2\sqrt{t}}$

$t^{\frac{1}{2}+\frac{2}{2}}$

$V'(t) = 24t - \frac{1}{2\sqrt{t}}$

d) Find the position function s(t) of the object at any time t, where s(0) =1.

$s(t) = \int v(t)\,dt$

$s(t) = 4t^3 - \frac{2}{3} t^{3/2} + 1$

$\frac{2t^{\frac{3}{2}}}{3}$

$\int v(t)\,dt = 12\frac{t^3}{3} - \frac{2}{3} t^{3/2} + 1$

e) Find the maximum and the minimum velocity of the object on the interval [0, 1].

$V(0) = 4$

$V(1) = 15$

f) Find the average velocity of the object on the interval [0, 1].

$\frac{s(1) - s(0)}{1} = \frac{-5}{3} - 1 = \frac{-5}{3} - \frac{3}{3} = \boxed{\frac{-8}{3}}$

(10 points) 11. a) Sketch the regions bounded by the x-axis and the graph of $y = x^2 - 2x - 3$ on the interval from x=0 to x=5.

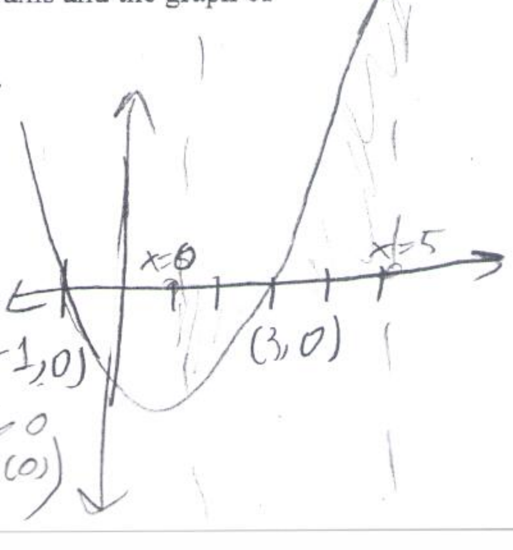
b) Label the intercepts and the point(s) of intersection.

c) Then find the total area of the bounded regions.

$\int_0^5 (x^2 - 2x - 3)\,dx$

$\frac{x^3}{3} - \frac{2x^2}{2} - 3x \Big|_0^5 = \boxed{\frac{5}{3}}$

$\left(\frac{(5)^3}{3} - (5)^2 - 3(5)\right) - \left(\frac{(0)^3}{3} - (0)^2 - 3(0)\right)$



3-3

(12 points) 10. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion.

a) State the limitations that are placed on variables t and V in context to the problem.

t can't be negative

V can't exceed speed of light

b) Find the initial velocity of the object.

$V(0) = 12(0) - \sqrt{0} + 4$

$\boxed{V(0) = 4}$

c) Find the acceleration function a(t) of the object at any time t.

$V'(t) = a(t)$

$V(t) = 12t^2 - t^{1/2} + 4$

$\boxed{a(t) = 24t - \frac{1}{2}t^{-\frac{1}{2}}}$

d) Find the position function s(t) of the object at any time t, where s(0) =1.

$V(t) = 12t^2 - \sqrt{t} + 4$ $\boxed{s(t) = 4t^3 - \frac{2t^{3/2}}{3} + 4t + 1}$

$s(t) = \int (12t^2 - t^{1/2} + 4)dt$

$s(t) = \frac{12t^3}{3} - \frac{t^{3/2}}{3/2} + 4x + C$

$s(t) = 4t^3 - \frac{2t^{3/2}}{3} + 4t + C$

e) Find the maximum and the minimum velocity of the object on the interval [0, 1].

$y = x^2+3$
$y' = 2x = 0$
$x = 0$
$= 0$

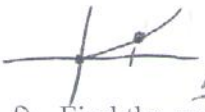
$V(t) = 12t^2 - t^{\frac{1}{2}} + 4$

$a(t) = 24t - \frac{1}{2\sqrt{t}} = 0 = \frac{24t}{1} - \frac{1}{2\sqrt{t}} \Rightarrow \frac{48t^{3/2}}{2\sqrt{t}} - \frac{1}{2\sqrt{t}} = 0 \Rightarrow 48t^{3/2} - 1 = 0$

$48t^{3/2} = 1$

$t^{3/2} = \frac{1}{48}$

$t = \sqrt[3]{\left(\frac{1}{48}\right)^2}$



min @ x=0
max @ x=1

f) Find the average velocity of the object on the interval [0, 1].

$V(t) = 12t^2 - \sqrt{t} + 4$

$\int_0^1 (12t^2 - t^{\frac{1}{2}} + 4)$

$4t^3 - \frac{2t^{3/2}}{3} + 4x + C \Big|_0^1$

$\frac{F(1) - F(0)}{1 - 0} = \frac{\frac{22}{3}}{1} = \boxed{\frac{22}{3}}$

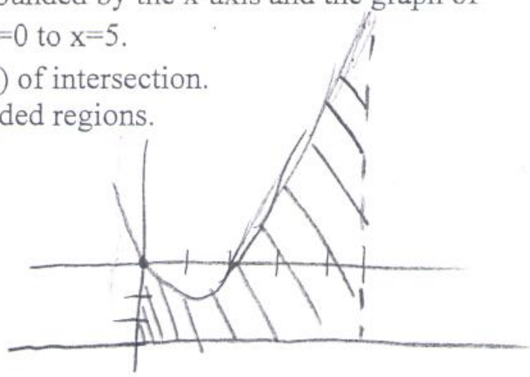
(10 points) 11. a) Sketch the regions bounded by the x-axis and the graph of $y = x^2 - 2x - 3$ on the interval from x=0 to x=5.

b) Label the intercepts and the point(s) of intersection.

c) Then find the total area of the bounded regions.

$\int (a-b)dx$ a → upper
b → lower

$\int_0^5 x^2 - 2x - 3$ $y = x^2 - 2x$ → upper
$y = 3$ → lower



(12 points) 10. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion.

a) State the limitations that are placed on variables t and V in context to the problem.

$t \geq 0$ V can be any real number

b) Find the initial velocity of the object.

$V(t) = 12(0) - \sqrt{0} + 4$ $V_i = 4$

c) Find the acceleration function a(t) of the object at any time t.

$a(t) = [V(t)]'$

$a(t) = 24t - \frac{1}{2\sqrt{t}}$

d) Find the position function s(t) of the object at any time t, where s(0) =1.

$\int 12t^2 - \sqrt{t} + 4\, dt = 4t^3 - \frac{2t^{3/2}}{3} + 4t + 1$

e) Find the maximum and the minimum velocity of the object on the interval [0, 1].

0.070

x	0	∧	$\sqrt[3]{1/2304}$	0.5	1
		−	0	+	

$24t - \frac{1}{2\sqrt{t}} = 0$

$24t = \frac{1}{2\sqrt{t}}$

$24t \cdot 2\sqrt{t} = 1$

$48t^{3/2} = 1$

$t^{3/2} = 1/48$

$t = (1/48)^{2/3}$

$t = \sqrt[3]{1/2304}$

Min: $\sqrt[3]{1/2304} = x$ Max: $x = 1$

f) Find the average velocity of the object on the interval [0, 1].

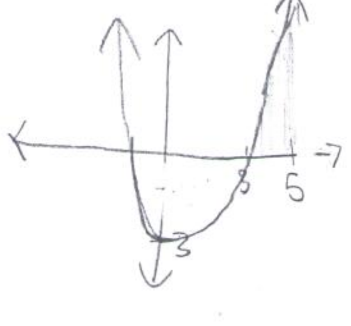
$12(1) - (1) + 4 = 15$

$\frac{15-4}{1} = 11$

(10 points) 11. a) Sketch the regions bounded by the x-axis and the graph of $y = x^2 - 2x - 3$ on the interval from x=0 to x=5.

b) Label the intercepts and the point(s) of intersection.

c) Then find the total area of the bounded regions.



$0 = (x-3)(x+1)$

$\frac{x^3}{3} - x^2 - 3x + C$

$\int_3^5 x^2 - 2x - 3 - 0\, dx$

$\frac{5^3}{3} - 25 - 15 = 1^2/_3$

$\frac{3^3}{3} - 9 - 9 =$

$1^2/_3 - (-9) = 10^2/_3$

(12 points) 10. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion.

a) State the limitations that are placed on variables t and V in context to the problem.

$t \nless 0$

b) Find the initial velocity of the object.

$12(0)^2 - \sqrt{0} + 4 = \underline{4}$

c) Find the acceleration function a(t) of the object at any time t.

$a(t) = \underline{24t - \frac{1}{2}t^{-1/2}}$

d) Find the position function s(t) of the object at any time t, where s(0) =1.

$\frac{12t^3}{3} - \frac{t^{3/2}}{3/2} + 4t + C \Rightarrow \underline{4t^3 - \frac{2t^{3/2}}{3} + 4t + 1}$

e) Find the maximum and the minimum velocity of the object on the interval [0, 1].

$\frac{\text{min} = 3.7936}{t = .0757}$ $\frac{\text{max} = 15}{t = 1}$

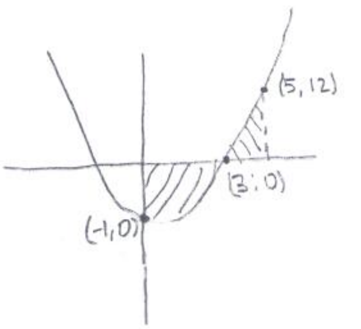
f) Find the average velocity of the object on the interval [0, 1].

$\frac{S_2 - S_1}{t_2 - t_1} \Rightarrow \frac{S(1) - S(0)}{1} = \underline{\frac{22}{3}}$

(10 points) 11. a) Sketch the regions bounded by the x-axis and the graph of $y = x^2 - 2x - 3$ on the interval from x=0 to x=5.

b) Label the intercepts and the point(s) of intersection.

c) Then find the total area of the bounded regions.



$\int_0^3 0 - (-x^2 - 2x - 3) + \int_3^5 -x^2 - 2x - 3 - 0$

$\Downarrow \qquad \Downarrow$

$\int_0^3 x^2 + 2x + 3 + \int_3^5 -x^2 - 2x - 3 \Rightarrow \frac{x^3}{3} + x^2 + 3x \Big|_0^3 + \frac{-x^3}{3} - x^2 - 3x \Big|_3^5$

$27 + \left(-\frac{164}{3}\right) = \boxed{-\frac{83}{3}}$

(12 points) 10. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion.

a) State the limitations that are placed on variables t and V in context to the problem.

$T \geq 0$ or cannot be negative

$V \geq \min$ or ≥ 0 mathematically

b) Find the initial velocity of the object.

$V(0) = 4$

c) Find the acceleration function a(t) of the object at any time t.

$a(T) = 24T - \frac{1}{2\sqrt{T}}$

d) Find the position function s(t) of the object at any time t, where s(0) = 1.

$\int 12t^2 - t^{\frac{1}{2}} + 4\, dT = 4t^3 - \frac{2}{3}T^{3/2} + 4T + 1$

e) Find the maximum and the minimum velocity of the object on the interval [0, 1].

$a = 24T - \frac{1}{2\sqrt{T}}$ $\quad 0 = 24T^{3/2} - \frac{1}{2}$ $\quad$ min $(0.0757, 3.7936)$ max $(1, 15)$

$a'(t) = 24 + \frac{1}{4x^{3/2}}$ $\quad \frac{1}{2} \cdot \frac{1}{24} = T^{3/2}$

$a'\left(\frac{1}{48^{2/3}}\right) = 24 + \frac{1}{(48^{2/3})^{3/2}} = +$ $\quad T^{3/2} = \frac{1}{48} = T^3 = \frac{1}{48^2}$. $\quad T \approx 0.0757$ or $\frac{1}{48^{2/3}}$

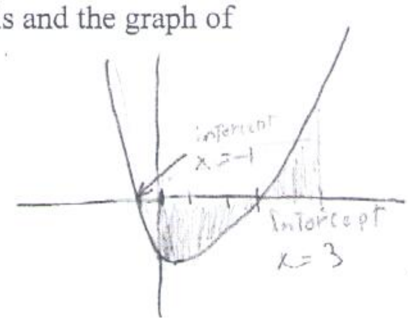
f) Find the average velocity of the object on the interval [0, 1].

$\frac{1}{1-0}\int_0^1 12t^2 - \sqrt{t} + 4\, dt = 4t^3 - \frac{2}{3}T^{3/2} + 4T\Big|_0^1 = 7.33$ or $\frac{22}{3}$

(10 points) 11. a) Sketch the regions bounded by the x-axis and the graph of $y = x^2 - 2x - 3$ on the interval from x=0 to x=5.

b) Label the intercepts and the point(s) of intersection.

c) Then find the total area of the bounded regions.



$-\int_0^3 x^2 - 2x - 3\, dx + \int_3^5 x^2 - 2x - 3\, dx$

$-\left(\frac{x^3}{3} - x^2 - 3x\right)\Big|_0^3 + \left(\frac{x^3}{3} - x^2 - 3x\right)\Big|_3^5$

$-(-9) + \frac{32}{3} = \frac{59}{3}$

(12 points) 10. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion.

a) State the limitations that are placed on variables t and V in context to the problem. V and t are only positive real numbers.

b) Find the initial velocity of the object.

$v(0) = 12(0)^2 - \sqrt{0} + 4 = 4$

initial velocity of 4.

c) Find the acceleration function a(t) of the object at any time t.

$v'(t) = a(t) = 24t - \frac{1}{2}t^{-1/2}$ $a(t) = 24t - \frac{1}{2\sqrt{t}}$

d) Find the position function s(t) of the object at any time t, where s(0) =1.

$s(t) = \int v(t)dt = \int 12t^2 - \sqrt{t} + 4\,dt$

$12\int t^2 - \int t^{1/2}dt + \int 4\,dt$ $s(t) = 4t^3 - \frac{2}{3}t^{3/2} + 4t + 1$

$\frac{12t^3}{3} - \frac{2t^{3/2}}{3} + 4t + C$

$4t^3 - \frac{2t^{3/2}}{3} + 4t + 1$

e) Find the maximum and the minimum velocity of the object on the interval [0, 1].

$a(t) = 0$ $24t - \frac{1}{2\sqrt{t}} = 0$ $24t = \frac{1}{2\sqrt{t}}$ $48t^{3/2} = 1$ min $= (0, 4)$ max $= (1,$

$48 = \sqrt{t^3}$

$2304 = t^3$ $t = \sqrt{2304}$ or 13.[illegible]

f) Find the average velocity of the object on the interval [0, 1].

$\frac{v(b) - v(a)}{b - a}$ $\frac{15 - 4}{1} = \frac{11}{1}$ = average velocity of 11 units/time

$\frac{v(1) - v(0)}{1 - 0}$

$v(1) = 12(1)^2 - \sqrt{1} + 4 = 11 + 4 = 15$

(10 points) 11. a) Sketch the regions bounded by the x-axis and the graph of $y = x^2 - 2x - 3$ on the interval from x=0 to x=5.

b) Label the intercepts and the point(s) of intersection. Total area of bounded regions. $5/3$ units^2

c) Then find the total area of the bounded regions.

$\int_0^5 x^2 - 2x - 3\,dx$

$\int x^2 dx - 2\int x\,dx - \int 3\,dx$

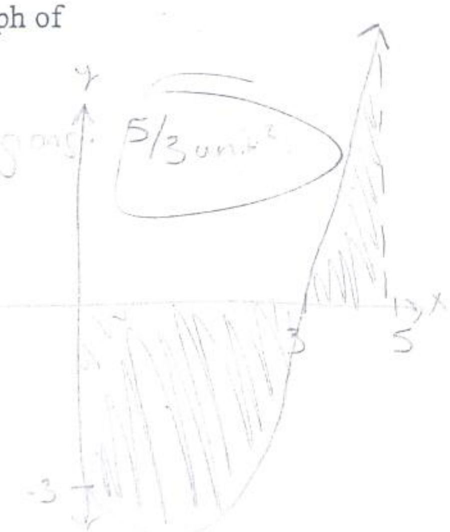
$F(x) = \frac{x^3}{3} - x^2 - 3x + C$

$F(b) - F(a) = \left(\frac{5^3}{3} - 25 - 15\right) - \left(\frac{0^3}{3} - 0 - 0\right)$

$= \frac{125}{3} - 40 = 5/3$

y-intercept at (0, -3)

x-intercept at (3, 0)



4-2

$V'(t) = 24t - \frac{1}{2\sqrt{t}}$

(12 points) 10. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion.

a) State the limitations that are placed on variables t and V in context to the problem.

t is limited to $(0, \infty)$ and V is limited to $(4, \infty)$

b) Find the initial velocity of the object.

$V(0) = 4$

c) Find the acceleration function a(t) of the object at any time t.

$a(t) = 24t - \frac{1}{2\sqrt{t}}$

d) Find the position function s(t) of the object at any time t, where s(0) =1.

$s(t) = -16t^2 - vt + 1$

e) Find the maximum and the minimum velocity of the object on the interval [0, 1].

$48t^{\frac{3}{2}}$

maximum : 15

minimum : 4

f) Find the average velocity of the object on the interval [0, 1].

$\frac{12t^3}{3} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 4x$

$\frac{\int_0^1 12t^2 - \sqrt{t} + 4}{1 - 0} = \frac{22}{3} = 7.\overline{33}$

(10 points) 11. a) Sketch the regions bounded by the x-axis and the graph of $y = x^2 - 2x - 3$ on the interval from x=0 to x=5.

b) Label the intercepts and the point(s) of intersection.

c) Then find the total area of the bounded regions.

$0 = (x-3)(x+1)$

$x = 3 \quad x = -1$

$\int_0^5 x^2 - 2x - 3$

$= F(5) - F(0) = \boxed{\frac{32}{3}}$

4-3

(12 points) 10. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion.

a) State the limitations that are placed on variables t and V in context to the problem.

t and V must be positive real numbers.

b) Find the initial velocity of the object.

$v(0) = 12(0)^2 - \sqrt{0} + 4 = 0 + 0 + 4 = 4$

c) Find the acceleration function a(t) of the object at any time t.

$a(t) = V'(t)$

$a(t) = 24t - \frac{1}{2\sqrt{t}}$

d) Find the position function s(t) of the object at any time t, where s(0) =1.

$s(t) = \int v(t)\,dt = \int 12t^2 - \sqrt{t} + 4\,dt$

$s(t) = 4t^3 - \frac{2}{3}t^{3/2} + 4t + 1$

e) Find the maximum and the minimum velocity of the object on the interval [0, 1].

Critical points: $x = 0$, $x = \frac{1}{48^{2/3}}$

x	0	0	$\frac{1}{48^{2/3}}$	1
f'(x)		ϕ −	ϕ +	

0 is a local max,
$\frac{1}{48^{2/3}}$ is a local min

f) Find the average velocity of the object on the interval [0, 1]

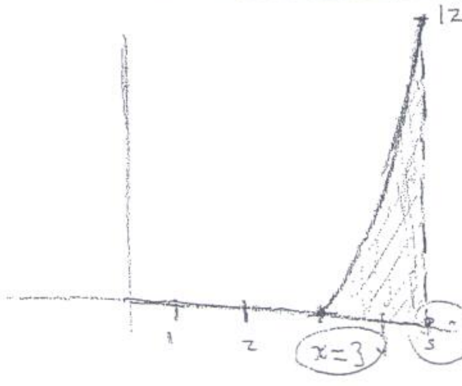
$\frac{s(1) - s(0)}{1 - 0} = \frac{(4(1)^3 - \frac{2}{3}(1)^{3/2} + 4(1) + 1) - (4(0)^3 - \frac{2}{3}(0)^{3/2} + 4(0) + 1)}{1}$

$= \frac{22}{3}$

(10 points) 11. a) Sketch the regions bounded by the x-axis and the graph of $y = x^2 - 2x - 3$ on the interval from x=0 to x=5.

b) Label the intercepts and the point(s) of intersection.

c) Then find <u>the total area</u> of the bounded regions.



x intercepts: $x = 3$, $x = -1$

y intercepts: $y = -3$

points of intersection: $(3,0), (-1,0)$

$\int_0^5 x^2 - 2x - 3\,dx$

$\int_0^5 x^2\,dx - \int_0^5 2x\,dx - \int_0^5 3\,dx$

$\frac{1}{3}x^3\Big|_0^5 - x^2\Big|_0^5 - 3x\Big|_0^5 = 5/3$

4-4

(12 points) 10. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion.

a) State the limitations that are placed on variables t and V in context to the problem.

t must be positive $(t \geq 0)$

$V \neq 0$

b) Find the initial velocity of the object.

$V(0) = 12(0)^2 - \sqrt{0} + 4 = \boxed{4}$

c) Find the acceleration function a(t) of the object at any time t.

$a(t) = V'(t)$

$a(t) = 24t - \frac{1}{2t^{1/2}}$

d) Find the position function s(t) of the object at any time t, where s(0) =1.

$s(t) = \int 12t^2 - t^{1/2} + 4 \, dt = \frac{12t^3}{3} - \frac{2t^{3/2}}{3} + 4t + c$

$s(t) = 4t^3 - \frac{2t^{3/2}}{3} + 4t + 1$

$s(0) = 1$

e) Find the maximum and the minimum velocity of the object on the interval [0, 1].

$V'(t) = 24t - \frac{1}{2t^{1/2}} = 0$

$48t^{3/2} - 1 = 0$

$\frac{48t^{3/2} = 1}{48}$

$\rightarrow t^{3/2} = \frac{1}{48}$

$\boxed{t = \left(+\frac{1}{\sqrt{48}}\right)^3 \quad \text{L. min}}$

$V''(t) = 24 + \frac{1}{4t^{1/2}}$

$V''\left(+\frac{1}{\sqrt{48}}\right)^3 = 24 + \frac{1}{4\left(\frac{1}{\sqrt{48}}\right.}$

> 0 l. min

$\boxed{\text{Lmax DNE}}$

f) Find the average velocity of the object on the interval [0, 1].

(10 points) 11. a) Sketch the regions bounded by the x-axis and the graph of $y = x^2 - 2x - 3$ on the interval from x=0 to x=5.

b) Label the intercepts and the point(s) of intersection.

c) Then find the total area of the bounded regions.

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

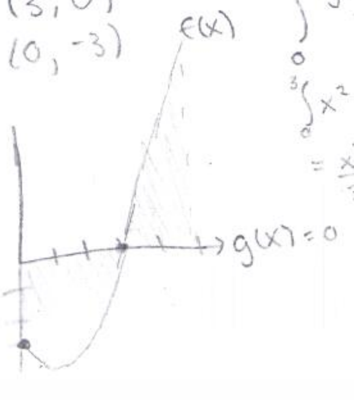
$x = 3 \quad \cancel{x=-1}$

$y = 0^2 - 2(0) - 3$

$y = -3$

(3, 0)

(0, -3)



$\int_0^3 g(x) - f(x) \, dx + \int_3^5 f(x) - g(x) \, dx$

$\int_0^3 x^2 - 2x - 3 \, dx + \int_3^5 x^2 - 2x - 3 \, dx$

$= \frac{x^3}{3} - x^2 - 3x \Big|_0^3 + \frac{x^3}{3} - x^2 - 3x \Big|_3^5$

$\frac{3^3}{3} - 3^2 - 3(3) - \left(\frac{0^3}{3} - 0^2 - 3(0)\right) + \left(\frac{5^3}{3} - 5^2 - 3(5)\right) - \left(\frac{3^3}{3} - 3^2 - 3(3)\right)$

$\cancel{9} - \cancel{9} - \cancel{9} + \frac{125}{3} - 25 - 15 - \cancel{-9})$

$= 5/3$

$A = \boxed{5/3}$

5-1

24. (12) Consider the velocity $v(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

(a) State the limitations that are placed on the variables t and v in context to the problem.
(b) Find the initial velocity of the object.
(c) Find the acceleration function $a(t)$ of the object at any time t.
(d) Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.
(e) Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.
(f) Find the average velocity of the object on the interval $[0, 1]$.

a) t has to be greater than 0

b)

~~c) a(t)~~

c) $a(t) = v'(t)$ $12t^2 - t^{\frac{1}{2}} + 4$

$a(t) = 24t - \frac{1}{2}t^{-\frac{1}{2}}$

d) $s(t) = \int 12t^2 - t^{\frac{1}{2}} + 4$ $\longrightarrow$ $s(t) = 4t^3 - \frac{2}{3}t^{\frac{3}{2}} + 4t + 1$

$\frac{12}{3}t^3 - \frac{1}{\frac{1}{2}+1}t^{\frac{1}{2}+1} + 4t + C$

$s(0) = 4t^3 - \frac{2}{3}t^{\frac{3}{2}} + 4t + C = 1$

$4(0)^3 - \frac{2}{3}(0)^{\frac{3}{2}} + 4(0) + C = 1$

$C = 1$

e) $12t^2 - t^{\frac{1}{2}} + 4 = 0$ $\quad 0 \leq t \leq 1$

$v(0) = 12(0)^2 - 0^{\frac{1}{2}} + 4 = 0$ $\quad$ max velo at $t = 1$

$v(1) = 12(1)^2 - 1^{\frac{1}{2}} + 4 = 15$ $\quad$ min velo at $t = 0$

f) $\frac{1}{1-0}\int_0^1 12t^2 - t^{\frac{1}{2}} + 4$

$4t^3 - \frac{2}{3}t^{\frac{3}{2}} + 4t \Big|_0^1$

$4(1)^3 - \frac{2}{3}(1)^{\frac{3}{2}} + 4(1) - 4(0)^3 - \frac{2}{3}(0)^{\frac{3}{2}} + 4(0) = 5.333$

5 - 2

24. (12) Consider the velocity $v(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

(a) State the limitations that are placed on the variables t and v in context to the problem.
(b) Find the initial velocity of the object.
(c) Find the acceleration function $a(t)$ of the object at any time t.
(d) Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.
(e) Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.
(f) Find the average velocity of the object on the interval $[0, 1]$.

$9 - 2/3$

a. $t \geq 0$

b. $v(0) = 12(0)^2 - \sqrt{0} + 4$

$v(0) = 4$

c. $24t - 1/2\, t^{-1/2}$

d. $\int 12t^2 - t^{1/2} + 4$

$4t^3 - 2/3\, t^{3/2} + 4t + C$

$1 = 4(0)^3 - 2/3\,(0)^{3/2} + 4(0) + C$

$1 = C$

$4t^3 - 2/3\, t^{3/2} + 4t + 1$

e.

f. $\dfrac{s(1) - s(0)}{1 - 0} = \dfrac{(4 - 2/3 + 4 + 1) - 1}{1}$

$= 25/3$

e. ~~$12t^2 - \sqrt{t} + 4 = 0$~~

~~$(12t^2 + 4)^2 = \sqrt{t}^2$~~

I Don't remember the algebra :($144t^4 + 96t^2 + 16 = t$

$f(1) = 15$

$f(0) = 4$

max @ $x=1$

minimum @ $x = 0$

5 - 3

24. (12) Consider the velocity $v(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

(a) State the limitations that are placed on the variables t and v in context to the problem.

(b) Find the initial velocity of the object.

(c) Find the acceleration function $a(t)$ of the object at any time t.

(d) Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.

(e) Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.

(f) Find the average velocity of the object on the interval $[0, 1]$.

a) $t > 0$, $v(t)$ is the velocity of the object in motion at a certain time t

b) $v(0) = 12(0)^2 - \sqrt{0} + 4 = 4$

c) $v'(t) = 24t - \frac{1}{2}t^{1/2} + 4 = a(t)$

d) $\int 12t^2 - t^{1/2} + 4 = 12\frac{t^3}{3} - \frac{2}{3}t^{3/2} + 4t + C = s(t)$

$s(0) = 12\frac{0^3}{3} - \frac{2}{3}(0)^{3/2} + 4(0) + c = 1$

$0 + C = 1$

$C = 1$

$s(t) = 12\frac{t^3}{3} - \frac{2}{3}t^{3/2} + 4t + 1$

e) $a(t) = 24t - \frac{1}{2}t^{1/2} + 4 = 0$

$24t - \frac{1}{2}t^{1/2} = -4$

$t(24 - \frac{1}{2}(1)^{-1/2}) = -4$

$t = \frac{-4}{24 - \frac{1}{2}1^{1/2}}$

$t = -\frac{8}{47}$

$v(t)$

$-8/47$

Critical point is not in domain so max and min must be at endpoints

+ +

0 1

relative min is at $x = 0$

relative max is at $x = 1$

f) $\frac{v(1) - v(0)}{2} = \frac{11}{2}$ avg velo is $11/2$

5 - 4

24. (12) Consider the velocity $v(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

(a) State the limitations that are placed on the variables t and v in context to the problem.
(b) Find the initial velocity of the object.
(c) Find the acceleration function $a(t)$ of the object at any time t.
(d) Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.
(e) Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.
(f) Find the average velocity of the object on the interval $[0, 1]$.

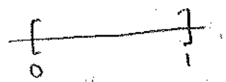
a) V(t) must be continuous & differentiable

b) $v(0) = 12(0)^2 - \sqrt{0} + 4 = \boxed{4}$

c) $a(t) = v'(t) = 24t - \frac{1}{2\sqrt{t}}$, $\boxed{a(t) = 24t - \frac{1}{2\sqrt{t}}}$

d) $s(t) = \int v(t) = 6t^3 - \frac{2}{3}t^{3/2} + 4t$, $\boxed{s(t) = 6t^3 - \frac{2}{3}t^{3/2} + 4t}$

e) $12(t)^2 - \sqrt{t} + 4 = 0$



$\min = 12(0)^2 - \sqrt{0} + 4 = \boxed{4}$

$\max = 12(1)^2 - \sqrt{1} + 4 = \boxed{15}$

f) $\frac{4 + 15}{2} = \boxed{9.5 \text{ avg velocity on } [0, 1]}$

$\frac{1}{t^2} \to -\frac{1}{2} t^{\frac{1}{2}}$

5-5

$24t$

24. (12) Consider the velocity $v(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

(a) State the limitations that are placed on the variables t and v in context to the problem.

(b) Find the initial velocity of the object.

(c) Find the acceleration function $a(t)$ of the object at any time t.

(d) Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.

(e) Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.

(f) Find the average velocity of the object on the interval $[0, 1]$.

(a) t must be ≥ 0, there can't be - time
v can be + or -, as it is a vector quantity

(b)
$v(0) = 12(0)^2 - \sqrt{0} + 4 = \boxed{v(0) = 4}$

the initial velocity is 4

(f)
$v(0)=4$ $v(1)=15$

average velocity ≈ 9.5

(c)
$a(t) = v'(t) = 24t - (-\frac{1}{2} t^{-\frac{1}{2}}) = 24t + \frac{1}{2} t^{-\frac{1}{2}}$

$\boxed{a(t) = 24t + \frac{1}{2} t^{-\frac{1}{2}}}$

(d)
$s(t) = \int v(t) = \int 12t^2 - \sqrt{t} + 4$

$\frac{12t^3}{3} - \frac{t^{\frac{3}{2}}}{\frac{1}{2}} \quad 4t$

$2t^{\frac{3}{2}}$

$s(t) = 4t^3 - 2t^{\frac{3}{2}} + 4t + C$

$s(0) = 1$

$4(0^3) - 2(0)^{\frac{3}{2}} + 0 + C = 1$

$C = 1$

$\boxed{s(t) = 4t^3 - 2t^{\frac{3}{2}} + 4t + 1}$

(e)
$a(t) = 0 = 24t + \frac{1}{2} t^{-\frac{1}{2}}$

$-24t = \frac{1}{2}\sqrt{t}$

$-24 \cdot 2 = \frac{\sqrt{t}}{t}$

$(-48)^2 = \left(\frac{\sqrt{t}}{t}\right)^2$

$2304 = \frac{t}{t^2}$

$2304 = \frac{1}{t}$

$t \cdot 2304 = 1$

$t = \pm\frac{1}{2304}$

the max is $\frac{1}{2304}$
the min is $-\frac{1}{2304}$

5 - 6

24. (12) Consider the velocity $v(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

(a) State the limitations that are placed on the variables t and v in context to the problem.

(b) Find the initial velocity of the object.

(c) Find the acceleration function $a(t)$ of the object at any time t.

(d) Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.

(e) Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.

(f) Find the average velocity of the object on the interval $[0, 1]$.

a.) $V(t) \geq 0 \rightarrow t \geq 0$

$12t^2 - \sqrt{t} + 4 = 0 \rightarrow -\sqrt{t} = t^{-\frac{1}{2}} = -\frac{1}{2}t^{-\frac{1}{2}-1} = -\frac{1}{2}t^{-\frac{3}{2}}$

$12t^2 = 0$ $\quad -\sqrt{t} + 4 = 0$ $\quad 12t^2 - \sqrt{t} + 4 = 0$

$t = 0$ $\quad (-\sqrt{t})^2 = (-4)^2$ $\quad -4 \quad -4$

$t = 16$

b.) $12(0)^2 - \sqrt{0} + 4$

initial $V(t) = 4$

c.) $24t - \frac{1}{2}t^{-\frac{3}{2}} + 0 = a(t)$

d.) $\int 12t^2 - \sqrt{t} + 4\,dt \rightarrow (12)\frac{1}{3}t^3 - \frac{1}{-\frac{1}{2}+1}t^{-\frac{1}{2}+1} + 4x = \frac{12}{3}t^3 - 2t^{\frac{1}{2}} + 4t + C$

$2\sqrt{t}$

$= \frac{1}{\frac{1}{2}}$ $\quad 2$

$S(t) = 4t^3 - 2\sqrt{t} + 4t + 1$

$S(0) = 1$

e.) maximum on $[0, 1] = 15$ $\quad V(0) = 4$

$V(1) = 15$

f.) avg $V(t) = 7$

$f_{avg} = \frac{1}{b-a}\int_a^b 12t^2 - \sqrt{t} + 4$ $\quad 4t^3 - 2t^{\frac{1}{2}} + 4x + 1$

$(1)\frac{7-0}{1-0} = 7$

5 - 7

24. (12) Consider the velocity $v(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

(a) State the limitations that are placed on the variables t and v in context to the problem.

(b) Find the initial velocity of the object.

(c) Find the acceleration function $a(t)$ of the object at any time t.

(d) Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.

(e) Find the maximum and the minimum velocity of the object on the interval $[0, 1]$.

(f) Find the average velocity of the object on the interval $[0, 1]$.

a) $t \geq 0$

b) Initial velocity when $t = 0$

$v(0) = 12(0^2) - \sqrt{0} + 4$

$v(0) = 4$

c) $a(t) = v'(t) = 24t - \dfrac{1}{2\sqrt{t}}$

d) $s(t) = \int v(t)\,dt$

$= \int 12t^2 - t^{1/2} + 4\,dt$

$= \dfrac{12t^3}{3} - \dfrac{t^{3/2}}{3/2} + 4t + c$

$= 4t^3 - \dfrac{2}{3}\sqrt{t^3} + 4t + c$

$s(0) = 4(0)^3 - \dfrac{2}{3}\sqrt{0^3} + 4(0) + c$

$1 = c$

$s(t) = 4t^3 - \dfrac{2}{3}\sqrt{t^3} + 4t + 1$

e) $v(1) = 12 - 1 + 4 = 15$

$v(0) = 4$

maximum velocity $= 15$

minimum velocity $= 4$

f) $f_{avg} = \dfrac{1}{1-0}\int_0^1 v(t)\,dt$

$= \left[4t^3 - \dfrac{2}{3}\sqrt{t^3} + 4t + 1\right]_0^1$

$= \left[4 - \dfrac{2}{3} + 4 + 1\right] - [1]$

$f_{avg} = \dfrac{22}{3}$