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MATH 136

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1-1

MATH 136 -1

Quiz 9 NAME:

Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

-V is a product of t and therefore it is dependent on t

b. Find the initial velocity of the object.

$V_i = t_0$ $V = 12(0)^2 - \sqrt{0} + 4$ $V=4$

c. Find the acceleration function a(t) of the object at any time t.

$(t)^{1/2}$

$a = V' = \frac{d}{dt}(12t^2 - \sqrt{t} + 4)$ $t=1$

$a(t) = 24t - \frac{1}{2\sqrt{t}}$ $24(1) - \frac{1}{2\sqrt{1}} = 24 - \frac{1}{2} = 24 \cdot 12$

$a = 12$

d. Find the position function s(t) of the object at any time t, where s(0) =1.

$12t^2 - \sqrt{t} + 4$ $\frac{12t^{2+1}}{3} - (t)^{1/2+1} + 4t$

$4t^3 - \frac{3}{2}t^{3/2} + 4t$ $4(0)^3 - \frac{3}{2}(0)^{3/2} + 4(0) = 0$

e. Find the maximum and the minimum velocity of the object on the interval [0, 1].

$12t^2 - \sqrt{t} + 4$

Max: 4

$12(0)^2 - \sqrt{0} + 4 = 4$

Min: 15

$12(1)^2 - \sqrt{1} + 4$

$12 - 1 + 4 = 15$

f. Find the average velocity of the object on the interval [0, 1].

$4 + 15 = 19$ $19/2 = 9.5$

1-2

MATH 136

Quiz 9

NAME:

Consider the velocity function V(t) = $12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

whether the t element is negative due to the $\sqrt{}$ and if the result of V is negative,

b. Find the initial velocity of the object.

$V(0) = 4$

c. Find the acceleration function a(t) of the object at any time t.

$A(t) = 24t - \frac{1}{2t^{\frac{1}{2}}}$

d. Find the position function s(t) of the object at any time t, where s(0) =1.

$s(t) = 4t^3 - \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + 4t$

$= 4 - \frac{2}{3} + 4 = \frac{22}{3}$

e. Find the maximum and the minimum velocity of the object on the interval [0, 1].

$V(0) = 4$ $V(1) = 15$

Min: 4 Max: 15

f. Find the average velocity of the object on the interval [0, 1].

$\frac{15 - 4}{1 - 0} = 11$

1-3

MATH 136

Quiz 9

NAME:

Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

"V is related to "t", & t is part of a function.

b. Find the initial velocity of the object.

$V(0) = 4$

c. Find the acceleration function a(t) of the object at any time t.

$V'(t) = \boxed{24t - \frac{1}{2t^{1/2}}}$

$t^{1/2} \to \frac{1}{2} t^{-1/2} \to \frac{1}{2t^{1/2}}$

d. Find the position function s(t) of the object at any time t, where s(0) =1.

$\boxed{\frac{12t^3}{3} - \frac{t^{3/2}}{3/2} + 4t}$

e. Find the maximum and the minimum velocity of the object on the interval [0, 1].

$\int_0^1 12t^2 - \sqrt{t} + 4 \to \frac{12t^3}{3} - \frac{t^{3/2}}{3/2} + 4t \Big|_0^1 = \boxed{\frac{22}{3}}$

f. Find the average velocity of the object on the interval [0, 1].

$\frac{\int f(x)\,dx}{(b-a)} = \frac{22/3}{1} = \boxed{\frac{22}{3}}$

1-4

MATH 136

Quiz 9

NAME:

Consider the velocity function V(t) = $12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

V is the velocity

t is time

V is a function of t

b. Find the initial velocity of the object.

t= 0

$V(0) = 12(0)^2 - \sqrt{0} + 4 = 4$

c. Find the acceleration function a(t) of the object at any time t.

$a(t) = v'(t)$

$v'(t) = 24t - \frac{1}{2}t^{-1/2} = 24t - \frac{1}{2\sqrt{t}} = a(t)$

d. Find the position function s(t) of the object at any time t, where s(0) =1.

$S(t) = \int (12t^2 - \sqrt{t} + 4)dt$

$= 12 \cdot \frac{t^3}{3} - \frac{t^{3/2}}{3/2} + 4t + C$

$= 4t^3 - \frac{2t^{3/2}}{3} + 4t + 1$

S(t) where $S(0) = 1$ $= 4t^3 - \frac{2t^{3/2}}{3} + 4t + 1$

e. Find the maximum and the minimum velocity of the object on the interval [0, 1].

critical #'s : $V'(t) = 0$ or DNE

$12t^2 - \sqrt{t} + 4 = 0$

min: $12(0)^2 - \sqrt{0} + 4 = 0$

max: $12(1)^2 - \sqrt{1} + 4 = 15$

f. Find the average velocity of the object on the interval [0, 1].

$\frac{1}{1-0}\int_0^1 12t^2 - \sqrt{t} + 4 = 1\left(4t^3 - \frac{2t^{3/2}}{3} + 4t\right)\Big|_0^1$

$= \left[4(1)^3 - \frac{2(1)^{3/2}}{3} + 4(1)\right] - \left[4(0)^3 - \frac{2(0)^{3/2}}{3} + 0\right]$

$= 7.33$

7

2-1

5. (9 points) Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion in feet per second. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

On a graph where time is x, and velocity is y...
time begins at 0 and can go to ∞. So the domain is $[0, \infty]$.
velocity at time = 0 is 4, so the range is $[4, \infty]$.

b. Find the initial velocity of the object. Write a sentence in context of the problem, including the units, interpreting the initial velocity of the object.

$V(0) = 12(0)^2 - \sqrt{0} + 4$
$V(0) = 4$

at time $t = 0$, the velocity of the object is 4 feet/second.

c. Find the acceleration function a(t) of the object at any time t. Be sure to include the appropriate units.

$a(t) = v'(t) = 24t - \frac{1}{2}t^{-1/2}$

$24t - \frac{1}{2}t^{-1/2} = a$ feet/second^2

d. Find the position function s(t) of the object at any time t, where s(0) =1. Be sure to include the appropriate units.

$s(t) = \int v(t)dt = \int 12t^2 - \sqrt{t} + 4 = 12\int t^2 - \int t^{1/2} + \int 4$

$= 12\left(\frac{t^3}{3}\right) - \left(\frac{t^{3/2}}{3/2}\right) + 4t = \frac{12t^3}{3} - \frac{2t^{3/2}}{3} + 4t = S$ feet

e. Find the maximum and the minimum velocity of the object on the interval [0, 1].

$v(0) = 4$ feet/second
$v(1) = 12(1)^2 - \sqrt{1} + 4 = 12 - 1 + 4 = 15$ feet/second

Minimum = (0, 4)
Maximum = (1, 15)

Can solve this way because range is known as $[4, \infty)$

f. Find the average velocity of the object on the interval [0, 1].

average $= \frac{1}{1-0}\int_0^1 12t^2 - \sqrt{t} + 4$

using fundamental theorem of Calculus

$= \left(\frac{1}{1}\right)\left[\frac{12t^3}{3} - \frac{2t^{3/2}}{3} + 4t\right]_0^1 = \left[\left(\frac{12(1)^3}{3} - \frac{2(1)^{3/2}}{3} + 4(1)\right) - \left(\frac{12(0)^3}{3} - \frac{2(0)^{3/2}}{3} + 4(0)\right)\right]$

$= \frac{12}{3} - \frac{2}{3} + 4 = \frac{22}{3}$ feet/second

2-2

5. (9 points) Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion in feet per second. Show all work clearly.

   a. State the limitations that are placed on variables t and V in context to the problem.

   V = velocity   t = time

   b. Find the initial velocity of the object. Write a sentence in context of the problem, including the units, interpreting the initial velocity of the object.

   c. Find the acceleration function a(t) of the object at any time t. Be sure to include the appropriate units.

   $24t - \frac{1}{2t^{\frac{1}{2}}}$

   d. Find the position function s(t) of the object at any time t, where s(0) =1. Be sure to include the appropriate units.

   $s(t) = 4t^3 - \frac{2}{3}t^{\frac{3}{2}} + 4t + 1$

   $1 = 4(0)^3 - \frac{2}{3}(0)^{\frac{3}{2}} + 4(0) + C$

   $1 = C$

   e. Find the maximum and the minimum velocity of the object on the interval [0, 1].

   max = (1, 15)

   min = DNE

   f. Find the average velocity of the object on the interval [0, 1].

2-3

5. (9 points) Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion in feet per second. Show all work clearly.

   a. State the limitations that are placed on variables t and V in context to the problem.

   height of dropped object

   b. Find the initial velocity of the object. Write a sentence in context of the problem, including the units, interpreting the initial velocity of the object.

   $\frac{d}{dx} = 24t - t^{\frac{1}{2}}$

   The initial velocity of the object is $V(t) = 24t - t^{\frac{1}{2}}$ ft/s.

   c. Find the acceleration function a(t) of the object at any time t. Be sure to include the appropriate units.

   $24t - t^{\frac{1}{2}} = 0$

   d. Find the position function s(t) of the object at any time t, where s(0) =1. Be sure to include the appropriate units.

   e. Find the maximum and the minimum velocity of the object on the interval [0, 1].

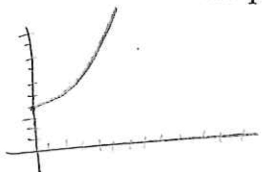
   f. Find the average velocity of the object on the interval [0, 1].

2-4

X 5. (9 points) Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion in feet per second. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

$$\boxed{\begin{array}{l} \text{domain} = (0, \infty) \\ \text{range} = [4, \infty) \end{array}}$$



← graph

b. Find the initial velocity of the object. Write a sentence in context of the problem, including the units, interpreting the initial velocity of the object.

$$v(0) = 12(0)^2 - \sqrt{0} + 4 = 4$$

An object is in motion, starting at 4 ft/sec, and increasing. The function $[12t^2 - \sqrt{t} + 4]$ describes the rate at which the object is moving.

c. Find the acceleration function a(t) of the object at any time t. Be sure to include the appropriate units.

$$v(t) = \left(12t^2 - \sqrt{t} + 4\right)' \Rightarrow 24t - t^{1/2} \Rightarrow 24t - \tfrac{1}{2}t^{3/2}$$

$$\boxed{a(t) = 24t - \left(\tfrac{1}{2}t^{3/2}\right) \text{ ft/sec}}$$

d. Find the position function s(t) of the object at any time t, where s(0) =1. Be sure to include the appropriate units.

$$s(t) = \int 12t^2 - \sqrt{t} + 4 = \frac{12t^3}{3} - \frac{2}{3}t^{3/2} + x + C$$

$$s(t) = \boxed{4t^3 - \tfrac{2}{3}t^{3/2} + x + C} \text{ ft/sec}$$

e. Find the maximum and the minimum velocity of the object on the interval [0, 1].

$$v(t) = 12t^2 - \sqrt{t} + 4$$

$$f(0) = 12(0)^2 - \sqrt{0} + 4 = 4$$

$$f(1) = 12(1)^2 - \sqrt{1} + 4 = 15$$

max	min
$f(1) = 15$	$f(0) = 4$

f. Find the average velocity of the object on the interval [0, 1].

$$\frac{1}{b-a} \text{ or } \frac{1}{1-0} = \boxed{1 \text{ ft/sec}}$$

Abigail Portmann

4-1

#8 (10 points of extra credit) Consider the velocity function $v(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and $v(t)$ in context to the problem.
b. Find the initial velocity of the object.
c. Find the acceleration function $a(t)$ of the object at any time t.
d. Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.
e. Find the maximum and the minimum velocity of the object on the interval [0,1].
f. Find the average velocity of the object on the interval [0,1].

a. time (t) can not be negative

b. initial velocity 4 $\quad 12(0)^2 - \sqrt{0} + 4 = 4$

c. $12t^2 - \sqrt{t} + 4 \rightarrow \boxed{24t - \frac{1}{2\sqrt{t}}}$

d. $4t^3 - \frac{2}{3}t^{\frac{3}{2}} + 4x + 1. \quad C = 1$

e. $12(0)^2 - \sqrt{0} + 4 = 4$ minimum $\quad 24t - \frac{1}{2\sqrt{t}} = 0$
$12(1)^2 - \sqrt{1} + 4 = 15$ maximum

f. $\frac{15 - 4}{1 - 0} = 11 \quad \frac{\frac{25}{3} - 1}{1 - 0} = \frac{22}{3}$

Tristam Burks
4-2

#8 (10 points of extra credit) Consider the velocity function $v(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and $v(t)$ in context to the problem.
b. Find the initial velocity of the object.
c. Find the acceleration function $a(t)$ of the object at any time t.
d. Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.
e. Find the maximum and the minimum velocity of the object on the interval [0,1].
f. Find the average velocity of the object on the interval [0,1].

a.) $t \geq 0$

time can't be negative

b.) $12t^2 - \sqrt{t} + 4 = V(0)$

$V(0) = 4$

c) $a(t) = 24t - \frac{1}{2}t^{-\frac{1}{2}}$

d.) $V(t) = 12t^2 - \sqrt{t} + 4$ $\quad S(0) = 1$

$S(t) = 4t^3 - 1\frac{1}{2}t^{1\frac{1}{2}} + 4t + C$

$1 = 4(0)^3 - 1\frac{1}{2}0^{1\frac{1}{2}} + 4(0) + C$

$C = 1$

$S(t) = 4t^3 - 1\frac{1}{2}t^{1\frac{1}{2}} + 4t + 1$

e.) min $(0, 4)$
max $(1, 15)$

f.) $\dfrac{15 - 4}{1 - 0} = 11$

Lee Monroe
4-3

#8 (10 points of extra credit) Consider the velocity function $v(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a. State the limitations that are placed on variables t and $v(t)$ in context to the problem.
b. Find the initial velocity of the object.
c. Find the acceleration function $a(t)$ of the object at any time t.
d. Find the position function $s(t)$ of the object at any time t, where $s(0) = 1$.
e. Find the maximum and the minimum velocity of the object on the interval [0,1].
f. Find the average velocity of the object on the interval [0,1].

a) limitations:
time variable, t, cannot be negative. It must be a real number, $0 \to \infty$.
velocity can be negative or positive.

b) initial velocity of the fx'n.

$f'(x)$ $12t^2 - \sqrt{t} + 4 =$

$12(0)^2 - \sqrt{0} + 4 =$

4

Initial velocity = 4

c) Velocity = $f'(x)$
acceleration = $f''(x)$ or $s''(x)$

$12t^2 - \sqrt{t} + 4$

$24t - (\frac{1}{2}t^{-1/2}) + 4$

$24t \cdot - \frac{1}{2\sqrt{t}}$

acceleration = $s''(x)$

$\boxed{24t \cdot - \frac{1}{2\sqrt{t}}}$

d) position function:

$\frac{12t^3}{3} - \frac{t^{3/2}}{3/2} + 4t + C = 1$

$0 + 0 + 0 + C = 1$

$C = 1$

$\boxed{\frac{12t^3}{3} - \frac{t^{3/2}}{3/2} + 4t + 1}$

Integral =

$\frac{12t^3}{3} - \frac{t^{3/2}}{3/2} + 4t$

e)
maximum velocity: 15
minimum velocity: 4
max = $f(1)$
min = $f(0)$

f) average velocity = $\frac{\text{change in distance}}{\text{change in time}}$ $\frac{\Delta s}{\Delta t}$ $\frac{15-4}{1-0}$ $\frac{11}{1} = 11$

avg velocity = 11

5-1

5. (9 points) Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion in feet per second. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

$t > 0$

b. Find the initial velocity of the object. Write a sentence in context of the problem, including the units, interpreting the initial velocity of the object.

$V(0) = 12(0)^2 - \sqrt{0} + 4 = \boxed{4}$

The initial velocity is 4 feet per second.

c. Find the acceleration function a(t) of the object at any time t. Be sure to include the appropriate units.

$V'(t) = 12t^2 - t^{1/2} + 4 = \boxed{24t - \frac{1}{2}t^{-1/2}}$

d. Find the position function s(t) of the object at any time t, where s(0) =1. Be sure to include the appropriate units.

$\int(12t^2 - \sqrt{t} + 4)\,dx = \frac{12t^3}{3} - \frac{2t^{3/2}}{3} + 4t + C = 4t^3 - \frac{2}{3}t^{3/2} + 4t + C$

$1 = 4(0)^3 - \frac{2}{3}(0)^{3/2} + 4(0) + C$

$1 = C$

$\boxed{4t^3 - \frac{2}{3}t^{3/2} + 4t + 1 = S(t)}$

e. Find the maximum and the minimum velocity of the object on the interval [0, 1].

min = 0, 1

f. Find the average velocity of the object on the interval [0, 1].

5-2

5. (9 points) Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion in feet per second. Show all work clearly.

a. State the limitations that are placed on variables t and V in context to the problem.

Domain: $(0, \infty)$

Range: $(4, \infty)$

b. Find the initial velocity of the object. Write a sentence in context of the problem, including the units, interpreting the initial velocity of the object.

$V(0) = 12(t)^2 - \sqrt{t} + 4 = 4$

c. Find the acceleration function a(t) of the object at any time t. Be sure to include the appropriate units.

$v'(t) = 24t - \frac{1}{2}t^{-3/2}$

d. Find the position function s(t) of the object at any time t, where s(0) =1. Be sure to include the appropriate units.

$\frac{x^{3/2}}{3/2}$

$s(t) = \int (12t^2 - \sqrt{t} + 4) = \frac{12t^3}{3} - \frac{2}{3}x^{3/2} + 4x$

$4t^3 - \frac{2}{3}x^{3/2} + 4x$

e. Find the maximum and the minimum velocity of the object on the interval [0, 1].

$v(0) = 4$ min

$v(1) = 15$ max

f. Find the average velocity of the object on the interval [0, 1].

$\frac{1}{1-0} \int_0^1 12t^2 - \sqrt{t} + 4 \quad \Big|_0^1 \; 4(1)^3 - \frac{2}{3}(1)^{3/2} + 4(1)$

$\boxed{7.3 \text{ ft/s}}$

11-1

$3x - \frac{x^3}{3}$

(10 points) 12. Calculate $\int_0^1 (3 - x^2)dx$ using the formula

$$\int_a^b f(x)dx = lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(a + \frac{b-a}{n} i)$$

$a = 0$

$b = 1$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} 3 - \left(+\frac{1}{n} i\right)^2$$

$$\sum_{i=1}^{n} 3 - \left(\frac{1}{n} \cdot i\right)^2$$

$$\lim_{n\to\infty} \frac{1}{n} \cdot \left(3n \frac{1}{n^2}\right)$$

$$\sum_{i=1}^{n} 3 - \frac{1}{n^2}$$

$$\lim_{n\to\infty} 3 - \frac{1}{n^3}$$

$$\lim_{n\to\infty} 3 - \frac{1}{(\infty)^3} = 3 - 0 = 3$$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.
b) Find the initial velocity of the object.
c) Find the acceleration function a(t) of the object at any time t.
d) Find the position function s(t) of the object at any time t, where s(0) =1.
e) Find the maximum and the minimum velocity of the object on the interval [0, 1].
f) Find the average velocity of the object on the interval [0, 1].

a) t can not equal any negative number, which means the domain of V is all positive real numbers

b) $t^2 - \sqrt{t} = \frac{1}{3}$ $\quad t^2 - t^{1/2} = \frac{1}{3}$ $\quad t^2 - t^{1/2} - 1/3 = 0$

$24t - \frac{1}{2\sqrt{t}} = 0$

$48t\sqrt{t} - 1 = 0$

$48t^{3/2} = 1$

$t^{3/2} = 1/48$

$t = 0.0030070377$

$[0, 1]$

c) $a(t) = 24t - \frac{1}{2\sqrt{t}}$

d) $\int 12t^2 - \sqrt{t} + 4$

$s(t) = \frac{12t^3}{3} - \frac{2t^{3/2}}{3} + 4t + C$

$s(t) = 4t^3 - \frac{2t^{3/2}}{3} + 4t + C$

$s(t) = 4t^3 - \frac{2t^{3/2}}{3} + 4t + 1$

e)

local min: 0.00307...

local max: 1

x	0	0.002	0.0030070037, 0.15	1
f(x)		↘		↗
f'(x)		−	0	+
			min	

f) $\frac{f(b) - f(a)}{b - a} = \frac{15 - 4}{1} = 11$

11-2

$i = \frac{n(n+1)}{2}$ $\quad 1 = \frac{n^2}{2} + \frac{1}{2}$

$1 = \frac{n(n+1)}{2}$ $\quad \sqrt{\frac{1}{2}} = \sqrt{\frac{n^2}{2}}$ $\sqrt{\frac{1}{2}} = \frac{n}{\sqrt{2}} \cdot \sqrt{2}$

$1 = \frac{n^2+1}{2}$ $\quad 1 = n$

$a = 0$
$b = 1$

(10 points) 12. Calculate $\int_0^1 (3 - x^2)dx$ using the formula

$$\int_a^b f(x)dx = lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(a + \frac{b-a}{n} i)$$

$f(x) = 3 - x^2$

$\lim_{n\to\infty} \frac{1-0}{n} \sum_{i=1}^{n} f(0 + \frac{1-0}{n}(1))$

$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(\frac{1}{n} \cdot (1))$ $\quad \lim_{x\to\infty} \frac{1}{1} f(3-(1)^2)$

$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(\frac{1}{n})$ $\quad \lim_{x\to\infty} 1 \cdot f(2)$

$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(3 - (\frac{1}{n})^2)$ $\quad \lim_{x\to\infty} f(2) \Rightarrow 3 - 2^2$

$3 - 4$

$\boxed{= -1}$

$12t^2 - t^{\frac{1}{2}} + 4$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.
b) Find the initial velocity of the object.
c) Find the acceleration function a(t) of the object at any time t.
d) Find the position function s(t) of the object at any time t, where s(0) =1.
e) Find the maximum and the minimum velocity of the object on the interval [0, 1].
f) Find the average velocity of the object on the interval [0, 1].

a). Limitations for t is that it is never equal to zero
Limitations for V is it is never equal to 0

b). $12t^2 - \sqrt{t} + 4 = 0$ $\quad t(12t - 1^{\frac{1}{2}}) = -4$ $\quad 12t - 1^{\frac{1}{2}} = -4$

$12t^2 - t^{\frac{1}{2}} = -4$ $\quad t = -4$ $\quad 12t - 1 = -4$ $\quad t = -\frac{1}{4}$

$12t = -3$
$\overline{12}$

$V(-4) = 190$ $\quad V(-\frac{1}{4}) = -2.75$

c). $a(t) \Rightarrow v'(t) = \boxed{24t - \frac{1}{2} t^{-\frac{1}{2}}}$

d). $v(t) = s'(t) = \int 12t^2 - \sqrt{t} + 4 \Rightarrow 4t^3 - \frac{2}{3}t^{3/2} + 4t + C = s(t)$ $\quad s(0) = C$

$C = 1$ $\quad \boxed{s(t) = 4t^3 - \frac{2}{3}t^{3/2} + 4t + 1}$

e). $\boxed{v(0) = 4}$ $\boxed{v(1) = 15}$

f) $\frac{v(1) - v(0)}{}$ $= \frac{4-15}{} = \boxed{\bar{v} = -11}$

11-3

$a=0 \quad b=1$

(10 points) 12. Calculate $\int_0^1 (3-x^2)dx$ using the formula

$$\int_a^b f(x)dx = lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(a+\frac{b-a}{n} i)$$

$f(x)=3-x^2$

$\int_0^1 (3-x^2)\,dx \Rightarrow \frac{(3-x^2)^2}{2}\Big|_0^1 \quad 2-\frac{9}{2}=\boxed{-\frac{5}{2}}$

$\left(0+\frac{1-0}{n}i\right) \quad 3-\left(\frac{1}{n}i\right)^2 \quad 3-\left(\frac{i}{n}\right)^2 \quad \left(\frac{i}{n}\right)\left(\frac{i}{n}\right)$

$\lim_{n\to\infty} \frac{1-0}{n} \sum_{i=1}^{n} 3-\left(\frac{1}{n}i\right)^2 \Rightarrow \lim_{n\to\infty} \frac{1}{n} \cdot 3 \cdot \sum_{i=1} -\left(i^2 \frac{1}{n^2}\right)$

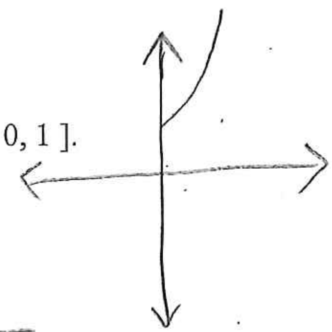
$\lim_{n\to\infty} \frac{1}{n} \cdot \Rightarrow 3-\frac{1}{n^2}\sum_{i=1}^{n} i^2 \quad \lim_{n\to\infty}\frac{1}{n} \cdot \Rightarrow 3-\frac{1}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right) \lim_{n\to\infty}\frac{1}{n} \Rightarrow 3-\frac{n(n+1)(2n+1)}{6n^2}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{n}\left(3-\frac{(n+1)(2n+1)}{6n}\right) = \lim_{n\to\infty} \frac{3}{n} - \frac{(n+1)(2n+1)}{6n^2} \Rightarrow \lim_{n\to\infty} \frac{18n(n+1)(2n}{6n^2}.$

$\Rightarrow \lim_{n\to\infty} \frac{3(n+1)(2n+1)}{n} \Rightarrow$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.
b) Find the initial velocity of the object.
c) Find the acceleration function a(t) of the object at any time t.
d) Find the position function s(t) of the object at any time t, where s(0) =1.
e) Find the maximum and the minimum velocity of the object on the interval [0, 1].
f) Find the average velocity of the object on the interval [0, 1].



b) Initial vel
$V(0) = \cancel{12(0)^2 - \sqrt{0}} + 4 = V(0) = 4$

$[0,1]$

e) $V(t) = 12(0)^2 - \sqrt{0} + 4$ min
$V(0) = 4$ min velocity
$V(t) = 12(1)^2 - \sqrt{1} + 4$
$12+3 = 15$ max veloci

c)
$a(t) = V'(t) = 24t - \frac{1}{2}t^{-\frac{1}{2}}$

f) $\text{average velocity} = \frac{(15+4)}{2} = 9.5$ avei velo

11-4

(10 points) 12. Calculate $\int_0^1 (3-x^2)dx$ using the formula

$a=0,\ b=1,\ f(x)=3-x^2$

$$\int_a^b f(x)dx = lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n} f(a+\frac{b-a}{n}i)$$

1) $f(0+\frac{i}{n}) \rightarrow f(\frac{i}{n}) \rightarrow 3-(\frac{i}{n})^2$

2) $\sum_{i=1}^{n} \frac{i^2}{n^2} = \frac{1}{n^2}\sum i^2 = \frac{1}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)$

3) $\lim_{n\to\infty} \frac{1}{n}\left(\frac{1}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)\right)$

$\lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3}$

$\lim_{n\to\infty} \frac{2n^2+3n+1}{6n^2}$

$\lim_{n\to\infty} \frac{1}{3} + \overset{0}{\frac{1}{2n}} + \overset{0}{\frac{1}{6n^2}} = \boxed{\frac{1}{3}}$

$\boxed{\int_0^1 (3-x^2)dx = \frac{1}{3}}$

$s(t) = \boxed{4t^3 - \frac{2t^{3/2}}{3} + 4x}$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.
b) Find the initial velocity of the object.
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d) Find the position function s(t) of the object at any time t, where s(0) =1.
e) Find the maximum and the minimum velocity of the object on the interval [0, 1].
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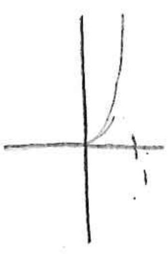


a) t must be positive, V can be pos/neg but must make sense to motion of object (pos/neg indicates obj. pos inc/dec)

b) $\boxed{V_0 = 4}$

c) $s''(t)$ $\boxed{a(t) = 24t - \frac{1}{2\sqrt{t}}}$

d) $s(t) = 4t^3 - \frac{2t^{3/2}}{3} + 4x + C$, $s(0) = 0 + C$, $C = 1$, $\boxed{s(t) = 4t^3 - \frac{2t^{3/2}}{3} + 4x + 1}$

*V is increasing, so I'll find V(a) for min & V(b) for max

e) $V(a) = 12(0) - 0 + 4 = 4$ $\quad V(b) = 12(1) - 1 + 4 = 15$

$\boxed{\text{Min velocity} = 4}$ $\quad \boxed{\text{max velocity} = 15}$

f) $\frac{\Delta v}{\Delta t} = \frac{V(1) - V(0)}{1-0} = \frac{(12(1)^2 - \sqrt{1} + 4) - (12(0)^2 - \sqrt{0} + 4)}{} = 12 + 3 - 4 = \boxed{9}$

11-5

(10 points) 12. Calculate $\int_0^1 (3-x^2)dx$ using the formula

$$\int_a^b f(x)dx = lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(a + \frac{b-a}{n} i)$$

$f(x) = 3 - x^2$

$\lim_{n\to\infty} \frac{1-0}{n} \sum_{i=1}^{n} 3 - (\frac{1}{n} i)^2$

$\int 3 - x^2 dx = 3x - \frac{1}{3}x^3 \Big|_0^1$

$= 3 - 1/3 - 0$

$= \frac{8}{3}$

$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} 3 + \sum_{i=1}^{n} -(\frac{1}{n} i)$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$\lim_{n\to\infty} 3 \sum_{i=1}^{n} -(\frac{1}{n} i)^2$

$\lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} i^2$

$\frac{3n^2 + 3n(2n+1) + 3n}{6n^3 + 6n^2 + 3n}$ L'H

$\lim_{n\to\infty} \frac{3n(n+1)(2n+1)}{6n} = \frac{\infty}{\infty} \xrightarrow{L'H} \frac{18n^2 + 18n + 3}{6}$

$\frac{3(6n^2 + 6n + 1)}{6} = \frac{6n^2 + 6n +}{3}$

$= \frac{8}{3}$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.
b) Find the initial velocity of the object.
c) Find the acceleration function a(t) of the object at any time t.
d) Find the position function s(t) of the object at any time t, where s(0) =1.
e) Find the maximum and the minimum velocity of the object on the interval [0, 1].
f) Find the average velocity of the object on the interval [0, 1].

$V(t) = 12t^2 - \sqrt{t} + 4$

a) - t is positive because it's time

- V is velocity & it's positive or negative depending on direction

c) $A(t) = v'(t) = 24t - \frac{1}{2}t^{-1/2}$

d) $S(t) = \int v(t)\,dt = 4t^3 - \frac{2}{3}t^{3/2} + 4x$

b) initial velocity $= 12(0)^2 - \sqrt{0} + 4 = 4$

f) average velocity

$= \frac{15 - 4}{1 - 0} = 11$

e) minimum velocity $= 4$

maximum velocity $= 15$

$24t - \frac{1}{2}t^{-1/2} = 0$

$24t - \frac{1}{2\sqrt{t}} = 0$

$24t = \frac{1}{2\sqrt{t}}$

$48t = \frac{1}{\sqrt{t}}$

$2304 = 1/t$

$t = 2304$

11-6

(10 points) 12. Calculate $\int_0^1 (3-x^2)\,dx$ using the formula

$$\int_a^b f(x)dx = lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n} f(a+\frac{b-a}{n}i)$$

$$\int_0^1 (3-x^2)\,dx = \lim_{n\to\infty} \frac{1-0}{n} \sum_{i=1}^{n} \left(3-\left(0+\frac{1-0}{n}i\right)^2\right)$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(3-\left(\frac{i}{n}\right)^2\right)$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(3-\frac{i^2}{n^2}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} (3) - \sum_{i=1}^{n}\left(\frac{1}{n^2}(i^2)\right)$$

$$= \lim_{n\to\infty} \frac{1}{n}\left((3n) - \frac{1}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)\right)$$

$$= \lim_{n\to\infty} \frac{1}{n}\left(3n - \frac{2n^3+3n^2+n}{6n^2}\right)$$

$$= \lim_{n\to\infty} 3 - \frac{2n^3+3n^2+n}{6n^3}$$

$$= 3 - \frac{1}{3}$$

$$= \boxed{\frac{8}{3}}$$

$n^2 + n$

$2n$ | $2n^3$ | $+2n^2$
$+1$ | n^2 | n

Check work:

$$\int_0^1 (3-x^2)dx = 3x - \frac{1}{3}x^3 \Big|_0^1$$

$$\overset{FTC-I}{=} 3 - \frac{1}{3} - 0 - 0$$

$$= 3 - \frac{1}{3}$$

$$= \frac{8}{3}$$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.
b) Find the initial velocity of the object.
c) Find the acceleration function a(t) of the object at any time t.
d) Find the position function s(t) of the object at any time t, where s(0) =1.
e) Find the maximum and the minimum velocity of the object on the interval [0, 1].
f) Find the average velocity of the object on the interval [0, 1].

a) t and V must be greater than or equal to 0. ($t \geq 0$ and $V \geq 0$)

b) Initial velocity = $V(0) = 4$

c) $a(t) = 24t - \frac{1}{2\sqrt{t}}$

d) $s(t) = 4t^3 - \frac{2}{3}t^{\frac{3}{2}} + 4t + 1$

e) The maximum velocity of the object on the interval $[0, 1]$ is 15.
The minimum velocity of the object on the interval $[0, 1]$ is 4.

f) The average velocity of the object on the interval $[0, 1]$ is $\frac{22}{3}$.

$$\frac{s(1)-s(0)}{1}$$

$4(1)^3 - \frac{2}{3}(1)^{\frac{3}{2}} + 4(1) + 1 - 1$

$4 - \frac{2}{3} + 4$

$8 - \frac{2}{3} = \frac{22}{3}$

$12(0)^2 - \sqrt{(0)} + 4 = 4$

$12(1)^2 - \sqrt{(1)} + 4 = 15$

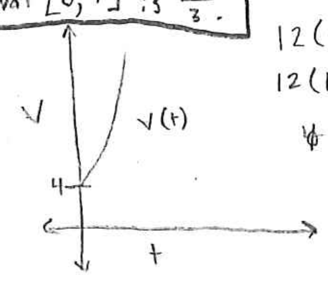
4 is min 15 is max

$V(0) = 12(0)^2 - \sqrt{(0)} + 4$

$a(t) = V'(t) = 24t - \frac{1}{2\sqrt{t}}$

$s(t) = \int 12t^2 - \sqrt{t} + 4\,dt = 4t^3 - \frac{2}{3}t^{\frac{3}{2}} + 4t + C$

$s(0) = 4(0)^3 - \frac{2}{3}(0)^{\frac{3}{2}} + 4(0) + C$



12 - 1

(10 points) 12. Calculate $\int_0^1 (3 - x^2)dx$ using the formula

$$\int_a^b f(x)dx = lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(a + \frac{b-a}{n} i)$$

Check:

$\int_0^1 (3-x^2)\,dx$

$\int_0^1 3\,dx - \int_0^1 x^2\,dx$

$3x\Big|_0^1 - \frac{x^3}{3}\Big|_0^1$

$3 - \frac{1}{3} = \frac{8}{3}$ FTC

1. $a=0$ $b=1$ $f(x) = 3-x^2$

2. $(0 + i\frac{1-0}{n}) = \frac{i}{n}$

3. $f(\frac{i}{n}) = 3 - \frac{i^2}{n^2}$

4. $\sum_{i=1}^{n} 3 - \frac{i^2}{n^2} \rightarrow \sum_{i=1}^{n} 3 \; \frac{-1}{n^2}\sum_{i=1}^{n} i^2$

$3n - \frac{1}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)$

$(n^2+n)(2n+1)$

$2n^3 + n^2 + 2n^2 + n$

5. $\lim_{n\to\infty} \frac{1}{n}\left(3n - \frac{1}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)\right)$

$3 - \frac{\cancel{2n^3} + \cancel{3n^2} + \cancel{n}}{6n^3} \rightarrow 3 - \frac{2}{6} \rightarrow \frac{18}{6} - \frac{2}{6} \rightarrow \frac{16}{6} \rightarrow \frac{8}{3}$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.
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e) Find the maximum and the minimum velocity of the object on the interval [0, 1].
f) Find the average velocity of the object on the interval [0, 1].

a)

b) $0 = 12t^2 - \sqrt{t} + 4$ $t =$

c) $a(t) = v'(t) = 24t - \frac{1}{2}(t)^{-1/2} + 0$

d) $s(t) = \int v(t) = 4t^3 - \frac{2}{3}t^{3/2} + 4x$

e)

f) $\frac{v(1) - v(0)}{1-0} \rightarrow \frac{15-4}{1} = 11$

12-2

(10 points) 12. Calculate $\int_0^1 (3-x^2)dx$ using the formula

FTC1

$3x - \frac{x^3}{3} \Big|_0^1$ $3-\frac{1}{3}$ 2.667 or $2\frac{2}{3}$

$$\int_a^b f(x)dx = lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n} f(a+\frac{b-a}{n}i)$$

$\frac{1-0}{n}$

$\int_0^1 f(x)$

$f(\frac{1}{n}i)$

$\int_0^1 f(x) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} 3 - \frac{1}{n}i^2$

$3n(2n^2+3n+1)$

$3\sum_{i=1}^{n}$ $\frac{n(n+1)(2n+1)}{6}$

$\frac{6n^3+9n^2+3n}{6} = 0$

$\frac{(n^2+n)(2n+1)}{6}$

$2n^3+n^2+2n^2+n$

$3\left(\frac{2n^3+3n^2+n}{6}\right)$

$= m \cdot a$

$\lambda = \frac{F}{m}$

$n=0$ $2n^2+3n+1=0$ $2n^2+3n=-1$ $= \boxed{\frac{8}{3}}$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.?

b) Find the initial velocity of the object. $24t + \frac{1}{2\sqrt{t}} = Vt$

c) Find the acceleration function a(t) of ~~the object at any time t.~~

d) Find the position function s(t) of the object at any time t, where s(0) =1. ?

e) Find the maximum and the minimum velocity of the object on the interval [0, 1]. ?

f) Find the average velocity of the object on the interval [0, 1]. ?

a.)

c.) $\frac{24t - \frac{1}{2\sqrt{t}}}{M}$

12 - 3

$a = 0 \quad b = 1 \quad f(x) = 3 - x^2$

(10 points) 12. Calculate $\int_0^1 (3 - x^2)dx$ using the formula

$$\int_a^b f(x)dx = lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(a + \frac{b-a}{n} i)$$

check w/ FTC-I:

$F(x) = 3x - \frac{x^3}{3}$

$\left[3(1) - \frac{(1)^3}{3}\right] - [0] = 3 - \frac{1}{3} = \boxed{\frac{8}{3}}$

① $0 + \frac{1}{n} i = \frac{i}{n}$

② $f\left(\frac{i}{n}\right) = 3 - \frac{i^2}{n^2}$

③ $\sum_{i=1}^{n} 3 - \frac{i^2}{n^2} = \sum_{i=1}^{n} 3 - \frac{1}{n^2} \sum_{i=1}^{n} i^2 = 3n - \frac{1}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right) = 3n - \frac{n(n+1)(2n+1)}{6n^2}$

④ $\frac{1}{n}\left(\frac{3n}{1} - \frac{n(n+1)(2n+1)}{6n^2}\right) = \frac{3\cancel{n}}{\cancel{n}} - \frac{\cancel{n}(\cancel{n+1})(\cancel{2n+1})}{6\cancel{n^3}}$

⑤ $\lim_{n\to\infty} 3 - \frac{2}{6} = 3 - \frac{1}{3} = \frac{9}{3} - \frac{1}{3} = \boxed{8/3}$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.
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e) Find the maximum and the minimum velocity of the object on the interval [0, 1].
f) Find the average velocity of the object on the interval [0, 1].

$\boxed{c.}$ $\boxed{a(t) = \frac{1}{2\sqrt{t}}}$

$\boxed{d.}$ $\int 12t^2 - \sqrt{t} + 4\, dt$

$\boxed{s(t) = 4t^3 - \frac{2\sqrt{t^3}}{3} + 4t + C}$

12-4

Check:

$3x - \frac{x^3}{3} \Big|_0^1$

$3 - \frac{1}{3} - 0 = \frac{8}{3}$ ✓

(10 points) 12. Calculate $\int_0^1 (3 - x^2)dx$ using the formula

$$\int_a^b f(x)dx = lim_{n\to\infty} \frac{b-a}{n}\sum_{i=1}^{n} f(a + \frac{b-a}{n} i)$$

1) $a = 0 \quad b = 1 \quad f(x) = 3 - x^2$

2) $(a + i\frac{b-a}{n}) = 0 + i\frac{1-0}{n} = i\frac{1}{n} = \frac{i}{n}$

3) $f(a + i\frac{b-a}{n}) = 3 - (\frac{i}{n})^2 = 3 - \frac{i^2}{n^2}$

4) $\sum_{i=1}^{n} f(a + i\frac{b-a}{n}) = \sum_{i=1}^{n} 3 - \frac{i^2}{n^2} = \sum_{i=1}^{n} 3 - \frac{1}{n^2}\sum_{i=1}^{n} i^2 = 3n - (\frac{1}{n}(\frac{n(n+1)(2n+1)}{6}))$

$= 3n - \frac{n(n+1)(2n+1)}{6n}$

5) $\lim_{n\to\infty} \frac{b-a}{n}\sum_{i=1}^{n} f(a + i\frac{b-a}{n}) = \lim_{n\to\infty} \frac{1}{n}(3n - \frac{n(n+1)(2n+1)}{6n}) = \lim_{n\to\infty}(3 - \frac{n(n+1)(2n+1)}{6n^2})$

$= 3 - \frac{2}{6} = \boxed{\frac{8}{3}}$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.
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d) Find the position function s(t) of the object at any time t, where s(0) =1.
e) Find the maximum and the minimum velocity of the object on the interval [0, 1].
f) Find the average velocity of the object on the interval [0, 1].

a) t must be positive b/c you cannot have a negative time in real life & their is a $\sqrt{t}$ in the velocity

V will always be positive due to t being squared,

b) $V(0) = 12(0)^2 - \sqrt{0} + 4 = \boxed{4}$

c) $a(t) = (12t^2 - \sqrt{t} + 4)' = \boxed{24t - \frac{1}{2\sqrt{t}}}$

d) $s(t) = \int 12t^2 - \sqrt{t} + 4 = 4t^3 - \frac{2t^{3/2}}{3} + 4t + C$

$s(0) = 4(0)^3 - \frac{2(0^{3/2})}{3} + 4(0) + C$

$s(0) = 0 + C$

$1 = C$

$\boxed{s(t) = 4t^3 - \frac{2t^{3/2}}{3} + 4t + 1}$

e) $V'(t) = 24t - \frac{1}{2\sqrt{t}} = 0$

$\frac{1}{2\sqrt{t}} = 24t$

$t\sqrt{t} = 0 \quad t\sqrt{t} = 0$

$t = 0$

$\boxed{\text{max} = 1 \quad \text{min} = 0}$

t	0	15	1
V'		+	
V		↗	

(f) $V(0) = 4 \quad V(1) = 15$

$V_{avg} = \frac{15 - 4}{0 - 1} = \boxed{-11}$

12/15

(10 points) 12. Calculate $\int_0^1 (3 - x^2)dx$ using the formula

$$\int_a^b f(x)dx = lim_{n\to\infty} \frac{b-a}{n}\sum_{i=1}^{n} f(a + \frac{b-a}{n} i)$$

1. $f(x) = (3 - x^2)$ $a = 0$ $b = 1$

2. $\frac{b-a}{n} = \frac{1}{n}$; $a + i\frac{b-a}{n} = \frac{i}{n}$

3. $f(\frac{i}{n}) = 3 - (\frac{i}{n})^2 = 3 - \frac{i^2}{n^2}$

4. $\sum_{i=1}^{n} 3 - \frac{i^2}{n^2} = \sum_{i=1}^{n} 3 - \sum_{i=1}^{n} \frac{i^2}{n^2} = 3n - \frac{1}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right) = 3n - \frac{(n+1)(2n+1)}{6n}$

5. $\lim_{n\to\infty} \frac{1}{n}\left[3n - \frac{(n+1)(2n+1)}{6n}\right] = \lim_{n\to\infty} \frac{3n}{n} - \frac{(n+1)(2n+1)}{6n^2} = \lim_{n\to\infty} 3 - \frac{2n^2+3n+1}{6n^2}$

$(n+1)(2n+1)$
$2n^2+n+2n+1$
$2n^2+3n+1$

$= 3 - \frac{1}{3}$

$= \boxed{\frac{8}{3}}$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.
b) Find the initial velocity of the object.
c) Find the acceleration function a(t) of the object at any time t.
d) Find the position function s(t) of the object at any time t, where s(0) =1.
e) Find the maximum and the minimum velocity of the object on the interval [0, 1].
f) Find the average velocity of the object on the interval [0, 1].

$12t^2 - t^{1/2} + 4$
$24t - \frac{1}{2}t^{-1/2}$

$\int 12t^2 - \sqrt{t} + 4 \, dt$
$4t^3 - \frac{2}{3}t^{3/2} + 4t + 1$

a) $t \geq 0$

b) initial velocity $= V(0) = 4$

c) $a(t) = V'(t)$

$a(t) = 24t - \frac{1}{2\sqrt{t}}$

d) $s(t) = \int V(t)\, dt$

$s(t) = 4t^3 - \frac{2}{3}t^{3/2} + 4t + 1$

) minimum $= 4$ f)

maximum $= 15$

12-6

$(n^2+n)(2n+1)$

$2n^3+3n^2+n$

(10 points) 12. Calculate $\int_0^1 (3-x^2)dx$ using the formula

$$\int_a^b f(x)dx = lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n} f(a+\frac{b-a}{n}i)$$

: $a=0$, $b=1$, $f(x)=3-x^2$

2: $0+\frac{1}{n}i = \frac{i}{n}$

3: $f(\frac{i}{n}) = 3-\frac{i^2}{n^2}$

1: $\sum_{i=1}^{n} 3 - \frac{1}{n^2}\sum_{i=1}^{n} i^2 = 3n - \frac{n(n+1)(2n+1)}{6n^2}$

5: $\lim_{n\to\infty} \frac{1}{n}\left(3n - \frac{n(n+1)(2n+1)}{6n^2}\right)$

$\lim_{n\to\infty}\left(\frac{3n}{n} - \frac{n(n+1)(2n+1)}{6n^3}\right)$

$= \lim_{n\to\infty} 3 - \lim_{n\to\infty}\frac{2n^3+3n^2+n}{6n^3}$

$= 3 - \lim_{n\to\infty}\frac{2\cancel{n^3}+3n^2\to 0+n\to 0}{6\cancel{n^3}}$

$= 3-\frac{1}{3}$

$\boxed{=\frac{8}{3}}$

Check: $\int_0^1 3-x^2\,dx$

$= \left[3x-\frac{x^3}{3}\right]\Big|_0^1$

$= 3-\frac{1}{3} = \frac{8}{3}$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.
b) Find the initial velocity of the object.
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f) Find the average velocity of the object on the interval [0, 1].

$t^{1/2} =$

$-t^{-1/2} = -\frac{1}{\sqrt{t}}$

) $D(f) = [0,\infty)$

$R(f) = [4,\infty)$

b) $V(0) = 4$

c) $a(t) = V'(t)$

$\therefore a(t) = 24t - \frac{1}{\sqrt{t}}$

) $s'(t) = V(t)$

$s(t) = \int 12t^2 - \sqrt{t} + 4$

$= \frac{12t^3}{3} - \frac{2t^{3/2}}{3} + 4t + C$

e) Min $= -\frac{b}{2a} = 0.1515$

$V(0.1515) = 3.8862$

Max $= V(1) = 12-1+4$

$= 15$

f) $= \frac{V(1)-V(0)}{1}$

$= \frac{15-4}{1}$

$= 9$

$1^2 \; n \; n(n+1)(2n+1)$
$(n^2+n)(2n+1)$
$2n^3+3n^2+n$

$F(x) = 3x - \frac{x^3}{3}\Big|_0^1$

(10 points) 12. Calculate $\int_0^1 (3-x^2)dx$ using the formula

$b=1 \quad a=0$
$f(x) = 3-x^2$

$$\int_a^b f(x)dx = lim_{n\to\infty} \frac{b-a}{n}\sum_{i=1}^{n} f(a+\frac{b-a}{n}i)$$

$\frac{b-a\,i}{n} = \frac{i}{n}$

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} 3-\left(\frac{i}{n}\right)^2 \Rightarrow \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} 3 - \frac{1}{n^2}\sum_{i=1}^{n} i^2 \Rightarrow$$

$$\lim_{n\to\infty} \frac{1}{n}\left[3n - \frac{2n^3+3n^2+n}{6n^2}\right] \Rightarrow \lim_{n\to\infty} 3 - \lim_{n\to\infty} \frac{2n^3+3n^2+n}{6n^3} \Rightarrow$$

$$3 - \lim_{n\to\infty} \frac{\cancel{n^3}(2+\cancel{3/n}^{\,0}+\cancel{1/n^2}^{\,0})}{\cancel{n^3}(6)} = 1/3 \Rightarrow 3 - 1/3 = \boxed{8/3 \text{ units}^2}$$

$s(t) = s'(t) = s''(t)$
$= V(t) = a(t)$

$\int \sqrt{t}\,dt = \int t^{1/2}dt = \frac{t^{3/2}}{3/2}$

(10 points) 13. Consider the velocity function $V(t) = 12t^2 - \sqrt{t} + 4$ of an object in motion. Show all work clearly.

a) State the limitations that are placed on variables t and V in context to the problem.
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f) Find the average velocity of the object on the interval [0, 1].

a) t can not be negative, as the square root would give imaginary.
b) $12(0)^2 - \sqrt{0} + 4 = 4$ initial velocity
c) $a(t) = 24t - \frac{1}{2\sqrt{t}}$ d) $4t^3 - \frac{2\sqrt{t^3}}{3} + 4t + 1$
e) max: $t=1$, $V(t) = 15$ min: $t=0$, $V(t) = 4$
f) avg: $\frac{15+4}{2} = \frac{19}{2} = 9.5$